

Photon phonon entanglement in coupled optomechanical arrays

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Abstract

We consider three optomechanical cavities coupled irreversibly as well as reversibly to each other and explore entanglement between the different optical and mechanical modes in the collective system. Each cavity is driven by a coherent field and the optomechanical coupling in each cavity is treated in the linearised regime. The composite system exhibits robust intercavity photon-phonon entanglement, in bipartite form, well into the steady state.

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I. INTRODUCTION

Quantum cavity optomechanics capitalizes on the radiation pressure of light exerted on a mechanical degree of freedom incorporated in confined high finesse cavities [1–3] to enable coupling between an optical and mechanical resonator. This currently active area of quantum physics is witnessing accelerated theoretical development [4–10] as well as experimental achievements [11–15]. Typically the cavity is driven to a steady state and the optomechanical coupling is linearised around the steady state amplitude. More recently, vacuum optomechanical coupling was studied [16]. Hence optomechanics serves as an excellent test bed for fundamental experiments at the quantum-classical boundary, paving innovative ways towards controlling the mutual interaction between light and the mechanical motion of mesoscopic objects. Sideband cooling to the absolute ground state of mechanical resonator systems [17–19] turned out to be quite a challenge experimentally, due to the detrimental but unavoidable thermal coupling of the resonators to their environments. Nevertheless these challenges have now been overcome, with different experimental groups recently having successfully achieved the ground state of mechanical oscillators in cavity electromechanical and optomechanical systems [20–23]. Hence we are now in the era of ground state mechanical systems and finally, the long anticipated quantum mechanical control of macroscopic objects is in sight. See [24] for an overview and outlook.

Akram et al. [25] recently showed how a discrete exchange of a non classical state, the single photon, can take place between the optical and mechanical modes of an optomechanical system, providing a feasible route towards quantum state transfer. In this paper we are interested in the dynamics of coupled optomechanical arrays. The mechanical and optical modes within one optomechanical system have been shown to exhibit a considerable degree of entanglement [26–29]. The entanglement between the output optical fields of a trapped-mirror-system has also been shown in [30]. A remarkable feature of optomechanical entanglement is that it can exist at temperatures above the absolute ground state. In [31], Mazzola and Paternostro consider the entanglement between a pair of optomechanical cavities, in the linearised regime, that arise when each cavity is driven by one of the twin beams generated by a source of spontaneous parametric down conversion. The optomechanical systems become entangled due to quantum correlations in the light sources driving each cavity. Here we explore the entanglement between optical and mechanical modes of three

different optomechanical cavities where they are coupled via their optical ports in either a reversible or irreversible (cascaded) manner. Another approach to coupled optomechanical arrays would be to couple the mechanical resonators rather than the optical resonators [32]. A system of two coupled microwave cavities each containing a mechanical element has also been considered by Heinrich and Marquardt [33].

II. THEORY AND MODEL

The basic idea in quantum cavity optomechanics is to induce a reversible coupling between an optical and mechanical resonator. Typically the interaction arises from the radiation pressure of light. The usual set up is modelled around an optical cavity whose resonance frequency is altered by the displacement of some mechanical resonator modelled as a harmonic oscillator. A shift in the resonance frequency of the optical cavity changes the circulating power and thus changes the radiation pressure on the mechanical resonator, yielding the optomechanical coupling which gives rise to a plethora of effects depending on how the various parameters and configurations are manipulated in the system. For example, the cavity can be driven to a steady state amplitude and the vacuum interaction linearised around this amplitude. This gives a coupling that is quadratic in the amplitude of the optical and mechanical resonator. The goal in recent experiments has been to push the macroscopic mechanical elements of optomechanical systems towards the quantum limit by various passive and active cooling protocols.

In this paper we consider a composite system of a chain of three identical cavities where the optical and mechanical resonators are quadratically coupled to each other. This can be accomplished in two different ways. Each cavity (1,2,3) is an optical Fabry-Perot cavity in which one of the mirrors is subject to a harmonic restoring force and can thus move due to radiation pressure, or with both mirrors fixed and an optically levitating nanosphere trapped within the optical cavity. Recently, optically levitating nanospheres have been proposed as new optomechanical systems with much higher Q factors due to their natural isolation from a thermal contact point [6–9] which is the most detrimental obstacle in achieving the pure ground state of the mechanical oscillator via optomechanical cooling. It is expected that these particular optomechanical systems will be much more efficiently cooled to their ground states as even at room temperature they are excellently well isolated [6].

The mechanical resonator in either model has a frequency $\omega_{m1,2,3}$ and damping rate $\mu_{1,2,3}$ while the optical cavity has a resonance frequency $\omega_{c1,2,3}$ and is strongly driven with a coherent pump field at frequency $\omega_{L1,2,3}$. Further, the trio of optomechanical cavities can be coupled together in two different configurations: via an **irreversible** coupling or a **reversible** coupling configuration as illustrated in Fig. 1.

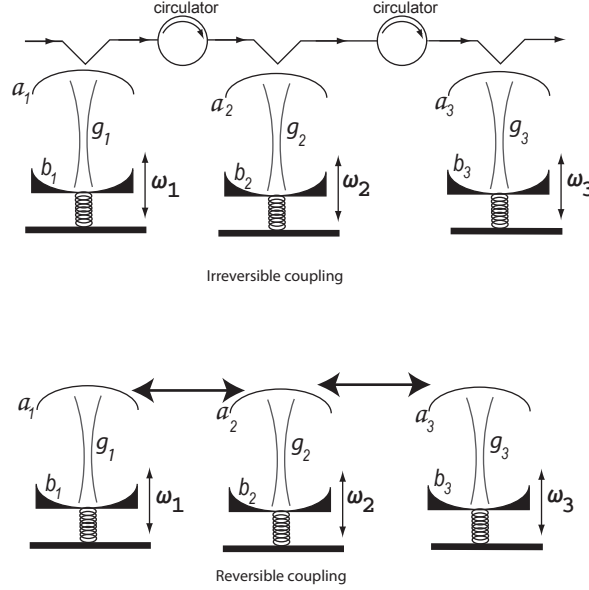


FIG. 1: Model: The three optomechanical cavities can be coupled either irreversibly via forward feed tunneling using circulators or reversibly via nearest neighbours.

The irreversible coupling is a forward feed method modelled as the cascaded systems approach [34]. Reversible coupling requires that the cavities be evanescently coupled and thus must be close to each other, however the irreversible (cascaded) coupling does not require this, but will require circulators be placed between the optical cavities to ensure irreversibility.

The Hamiltonian for the composite system is given as

$$H = \hbar \sum_{\alpha=1}^3 \Delta_{\alpha} a_{\alpha}^{\dagger} a_{\alpha} + \omega_{m_{\alpha}} b_{\alpha}^{\dagger} b_{\alpha} + g_{\alpha} (a_{\alpha} + a_{\alpha}^{\dagger}) (b_{\alpha} + b_{\alpha}^{\dagger}) \quad (1)$$

where a/b and a^{\dagger}/b^{\dagger} are the annihilation and creation operators respectively of each optical/mechanical mode and g is the optomechanical coupling rate within each cavity. Here we have considered the shifted reference frame where the amplitude of the cavity has been

displaced in the steady state due to the driving laser. Hence g is the effective coupling strength proportional to the steady state amplitude of the cavity field, due to linearisation of the radiation pressure force. $\Delta_\alpha = \omega_{c_\alpha} - \omega_{L_\alpha}$, is the detuning with respect to the driving laser field with $\alpha = 1, 2, 3$. The master equation for the composite system in the interaction picture with respect to the driving laser $\omega_{L_{1,2,3}}$ is given as,

$$\begin{aligned} \frac{d\rho}{dt} = & \frac{-i}{\hbar}[H, \rho] + \sum_{\alpha=1}^3 \kappa_\alpha \mathcal{D}[a_\alpha]\rho + \mu_\alpha(\bar{n} + 1)\mathcal{D}[b]\rho + \mu_\alpha\bar{n}\mathcal{D}[b_\alpha^\dagger]\rho \\ & + \mathcal{L}_{rev}\rho + \mathcal{L}_{irr}\rho \end{aligned} \quad (2)$$

where the reversible and irreversible dynamics are given respectively by

$$\mathcal{L}_{rev} = -i\chi_{12}[a_1a_2^\dagger + a_1^\dagger a_2, \rho] - i\chi_{23}[a_2a_3^\dagger + a_3^\dagger a_2, \rho] \quad (3)$$

$$\begin{aligned} \mathcal{L}_{irr} = & \sqrt{\kappa_1\kappa_2}([a_1\rho, a_2^\dagger] + [a_2, \rho a_1^\dagger]) + \sqrt{\kappa_1\kappa_3}([a_1\rho, a_3^\dagger] + [a_3, \rho a_1^\dagger]) \\ & + \sqrt{\kappa_2\kappa_3}([a_2\rho, a_3^\dagger] + [a_3, \rho a_2^\dagger]) \end{aligned} \quad (4)$$

We assume that the coupling is either reversible or irreversible but not both together. Here κ_j is the line width of each cavity- j , while χ_{kj} is an arbitrary coupling strength between the cavities k and j for the reversible coupling configuration and the damping superoperator \mathcal{D} is defined by

$$\mathcal{D}[A]\rho = A\rho A^\dagger - \frac{1}{2}(A^\dagger A\rho + \rho A^\dagger A) \quad (5)$$

As the cavities are driven by an external field, the coupling between the cavities is induced after each system attains steady state.

III. COUPLED OPTOMECHANICAL ARRAYS

A. Forward feed coupling

We first consider the forward feed coupling configuration between the three optomechanical cavities. Here the cavities are coupled via a unidirectional coupling only. In our calculations, we have neglected time delay between the cavities. Effectively, we can consider cavity 1 as the "main source" cavity while cavity 3 is only a receiver cavity. Cavity 2 on the other hand receives photons from cavity 1 as well as drives cavity 3. The topology of our set

up in this coupling configuration is further illustrated in Fig. 2. Hence we have reversible interactions between the optomechanical branches of the chain but irreversible interactions between the optical ports coupling the individual optomechanical units. This is quite a different configuration to previous work on coupled oscillator arrays [35].

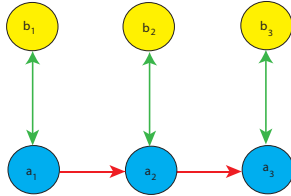


FIG. 2: Topology of chain of three optomechanical systems for forward feed (irreversible) coupling

As has been shown previously [29], intracavity photon-phonon entanglement is present within each optomechanical unit. However here we are interested in the presence of entanglement between the optical and mechanical resonators in distinct optomechanical cavities. Our composite system under study is in the Gaussian state as it starts from the vacuum, and the equations of motion are linear, we will have all Gaussian states in the system. Consequently, to quantify the entanglement, we use the logarithmic negativity measure for Gaussian states formulated by Vidal [36]. The log negativity between two states is expressed in terms of the entries of their covariance matrix, γ which is a 4×4 matrix given as

$$\gamma = \begin{pmatrix} \gamma_A & \gamma_C \\ \gamma_C^T & \gamma_B \end{pmatrix}, \quad \gamma_A, \gamma_B, \gamma_C \in M(2, \mathbb{R}) \quad (6)$$

The matrices $\gamma_{A,B}$ arise from position and momentum quadratures of the optical and mechanical modes respectively, while γ_C is a result of cross terms between the optical and mechanical position and momentum quadratures. The log negativity is then calculated as

$$E_N = \begin{cases} -\log f(\gamma)/2, & \text{if } f(\gamma) < 1, \\ 0, & \text{otherwise.} \end{cases} \quad (7)$$

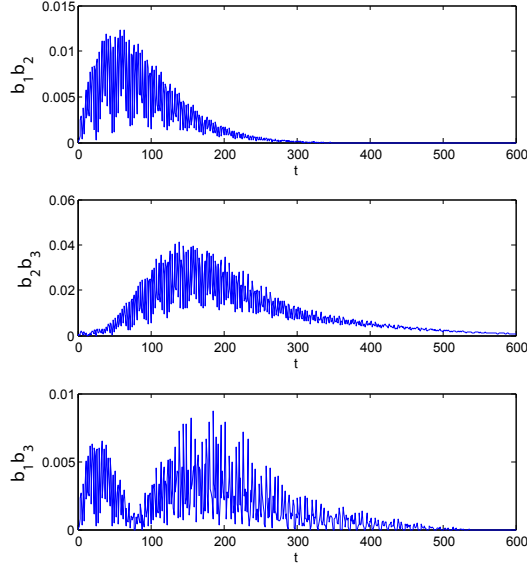


FIG. 3: *Forward feed coupling*

The log negativity between the intercavity phonons of the composite system of the three optomechanical units (b_1, b_2, b_3) vs. time for $\kappa_{1,2,3} = 0.1, \mu_{1,2,3} = 0.001, \Delta_{1,2,3} = \omega_{1,2,3} = 40, g_{1,2,3} = 0.5, \bar{n} = 0$.

where the function, $f : C(4) \rightarrow \mathbb{R}^+$ is defined as

$$f(\gamma) = \Gamma_{A,B,C} - \sqrt{\Gamma_{A,B,C}^2 - |\gamma|} \quad (8)$$

such that

$$\Gamma_{A,B,C} = \frac{|\gamma_A| + |\gamma_B|}{2} - |\gamma_C| \quad (9)$$

Hence in order to determine the matrix elements of each of the 2×2 matrix $\gamma_{A,B,C}$ we calculate the second order moments from the master equation of the composite system. Inserting the relevant second order moments into Eq. 7 we are now able to calculate the entanglement between any two modes of the system.

Fig. 3 shows the bipartite entanglement between all the mechanical mirrors b_1, b_2 and b_3 of the composite system for the feed forward coupling between the optical ports. We note the intercavity phonon-phonon entanglement exhibited by this system lasts well into the steady state.

Fig. 4 shows the entanglement between all possible pairs of the optical and mechanical modes of different cavities, i.e. intercavity photon-phonon entanglement. Clearly all the

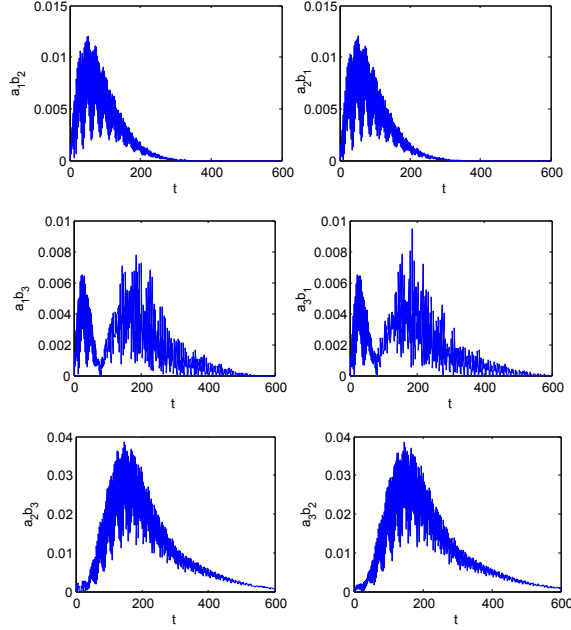


FIG. 4: *Forward feed coupling*

The log negativity between all possible selections of intercavity photons and phonons of the composite system of three optomechanical units vs. time for: $\kappa_{1,2} = 0.1, \mu_{1,2} = 0.001, \Delta_{1,2,3} = \omega_{1,2,3} = 40, g_{1,2,3} = 0.5, \bar{n} = 0$.

six modes of the system are shown to exhibit two mode bipartite entanglement within themselves. We note the symmetry for pairs of oscillators a_i, b_j and a_j, b_i . This is due to the symmetry in the Hamiltonian, Eq. 1 for cavities with identical parameters ω_{m_i} and g_i .

We have found the entanglement to be maximum when the detuning is resonant with the mechanical frequency, i.e. $\Delta = \omega_m$, hence the system is on the red sideband. We would like to remind the reader, that the full interaction Hamiltonian has been considered here, so that the squeezing terms $\hat{a}_i \hat{b}_i$ and $\hat{a}_i^\dagger \hat{b}_i^\dagger$ also contribute towards the dynamics of the system and in fact are responsible for the presence of the entanglement within each optomechanical unit. The traditional assumption, however would be to achieve strong entanglement over the blue sideband when $\Delta = -\omega_m$. But the amount of entanglement that could possibly be generated within each optomechanical unit is limited by the stability condition $g < \sqrt{\kappa\mu}/2$ arising in the Rotating Wave Approximation (RWA) limit, $\Delta = -\omega_m \gg \kappa, \mu$ [37]. However for optomechanical systems the mechanical damping rate μ is much less than the optical

damping rate, κ , consequently limiting the coupling strength g within each optomechanical unit to very small values ($g \approx 0.01$). As a result, for such weak coupling parameters, the intracavity entanglement does not transfer to intercavity modes on the blue sideband.

On the red sideband however, while the beam splitter interaction dominates over the squeezing interaction, stability conditions for $\Delta = \omega_m$, allow evolution of the system under the strong coupling regime, $g < \frac{1}{2}\sqrt{\omega_m^2 + \frac{\mu^2 + \kappa^2}{4}}$, taking the system away from the RWA limit [37]. This results in an exchange of excitations on the optomechanical branches of the chain of oscillators, consequently distributing the intracavity entanglement between oscillators of different cavities due to the forward feed coupling between the optical ports. It is interesting to see that photons and phonons of different cavities are robustly entangled without having been in direct contact with each other. Hence the forward feed coupling between the optomechanical units facilitates the distribution of intracavity entanglement over the intercavity modes.

Applying the relevant inequalities from Ref. [38], we determine that the multimode distribution of the entanglement we are observing in these results does not exhibit any genuine tripartite distribution. Hence we conclude that here our composite system of a chain of three optomechanical systems forms a class of entangled states of the optical and mechanical modes.

B. Reversible coupling

We now investigate the presence and possible distribution of entanglement between intercavity modes in the reversible coupling configuration. Here the three cavities are evanescently coupled with nearest neighbours with an arbitrary coupling strength χ_{ij} as given in Eq. 3. The topology we are considering here thus allows for a reversible exchange of optical excitations between the cavities as shown in Fig. 5.

In Fig. 6 we have the entanglement between the intercavity mechanical modes versus time. Clearly here also the mirrors of different optomechanical cavities are entangled for a long duration in the absence of a direct coupling induced between them.

Fig. 7 shows steady state intercavity photon-phonon entanglement between all two mode bipartite selections of the composite system of coupled optomechanical units. Again the system is limited by different stability conditions on the blue and red sidebands as discussed

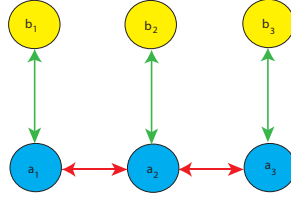


FIG. 5: Topology of chain of three optomechanical systems for reversible coupling

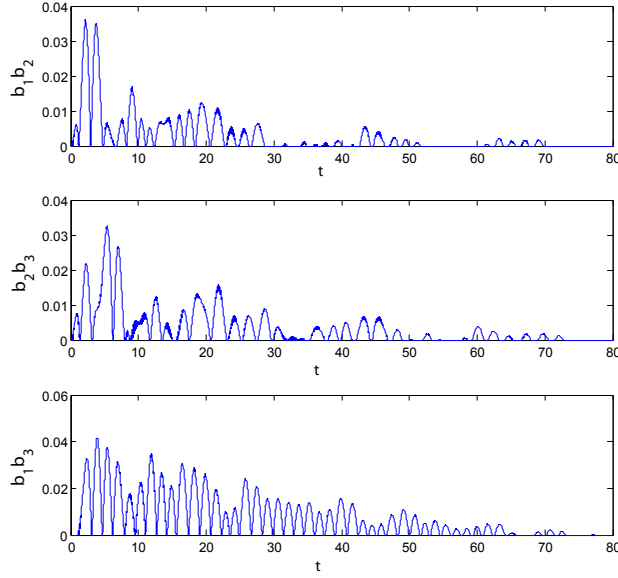


FIG. 6: *Reversible coupling*

The log negativity between the inter cavity phonons of the composite system of the three optomechanical units (b_1, b_2, b_3) vs. time for $\kappa_{1,2,3} = 0.1, \mu_{1,2,3} = 0.001, \Delta_{1,2,3} = \omega_{1,2,3} = 20, g_{1,2,3} = 1, \chi_{12} = \chi_{23} = 0.5, \bar{n} = 0$.

in section III A, so that we find the entanglement is distributed over intercavity modes only for the red sideband. Hence the intercavity entanglement is found to exist only on the red sideband of the system, as the beam splitter part of the Hamiltonian induces the distribution of the intracavity optomechanical entanglement over the cross cavity modes in the strong

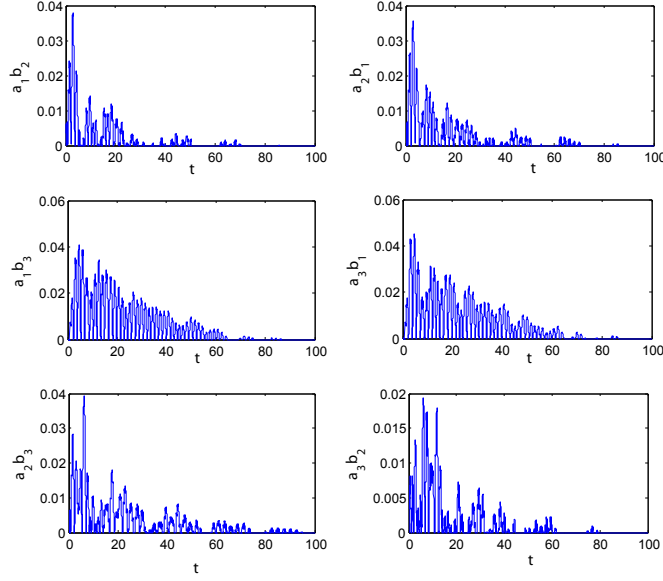


FIG. 7: *Reversible coupling*

The log negativity between all possible selections of inter cavity photons and phonons of the composite system of three optomechanical units vs. time for: $\kappa_{1,2,3} = 0.1, \mu_{1,2,3} = 0.001, \Delta_{1,2,3} = \omega_{1,2,3} = 20, g_{1,2,3} = 1, \chi_{12} = \chi_{23} = 0.5, \bar{n} = 0$.

coupling regime, satisfying the stability conditions of the system.

IV. SUMMARY

We have found that considerable robust steady state intercavity pair-wise entanglement exists between all the six modes $(a_{1,2,3}, b_{1,2,3})$ of a chain of three optomechanical systems when they are coupled both irreversibly via a feed forward as well as reversibly, via a nearest neighbour evanescent coupling. One would expect that the entanglement between the three cavity modes would exhibit some form of tripartite entanglement however, we find that applying the relevant inequalities from Ref. [38], the system of coupled optomechanical arrays does not exhibit genuine tripartite entanglement. Hence the system evolves such that pairs of oscillators are always accessible in two mode bipartite selections. Further, the topology of the chain of oscillators considered in this paper comprising of reversible optomechanical coupling between light and mechanics, alongside irreversible forward feed tunneling between optical ports, is quite distinct from earlier results on coupled oscillators.

We note that the oscillators are entangled for a longer duration in the irreversible coupling configuration as compared to the reversible case. The reversible case, based on nearest neighbour evanescent coupling, will require the cavities to be in close proximity, however the feedforward case is not so restricted. In this paper we have neglected delay effects in the feedforward case which means that delays must be less than the cavity decay times, which sets the time scale for the dynamics of the optomechanical array. Even with this assumption, the feedforward case enables the optical cavities to be much further apart — up to one metre — than the reversible coupling case. Coupled optomechanical arrays could have wide applications as components of quantum repeaters and quantum memories required in quantum information processing.

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